Title: Microeconomics' Assignment 2

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ECC4660 - Macroeconomics
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Assignment 2
Due: 17/5/2012
Hand in hard copy in class before 10:30am in order to get full credit
Show all the steps and relevant diagrams in order to get full credit

1. (20 points) Describe the business cycle model. Why is it important?

2. (19 points) What are the main differences between the business cycle models and the Keynesian models? If you were to give policy advice which framework would you use and why?

3. (36 points) Consider the Ramsey model with a representative household that lives forever and is endowed with one unit of time every period. This household derives utility from consumption and leisure given which is given by:
$$\sum_{t=0}^{\infty} \beta^t \left(\log(c_t) + \gamma \log(l_t)\right)$$
where 0 < β < 1 is the discount rate. There is a constant-return-to-scale production technology that allows firms to produce goods given by $F(K,L) = K^{\alpha} H^{1-\alpha}$ from K units of capital and H hours of labor. Firms must rent their capital and labor from households at the respective rental rates. The household owns all capital.
(a) Write down the representative household's budget constraint in a given period t, and formulate her utility maximization problem. (5 points)
(b) Find the equation that determines the consumption path. (7 points)
(c) Find conditions that characterize firms' profit maximization problem and market clearing conditions for the capital and labor market. (7 points)
(d) Derive the per capita capital accumulation for this economy. (7 points)
(e) What is the steady state per capita capital and consumption in this economy? (10 points)

4. (35 points) Consider a Diamond economy where young agents work, old retire, the production is Cobb-Douglas, utility is logarithmic in both young and old consumption and there is no population growth nor growth in the efficiency per worker. Suppose the government taxes each young person an amount τ and uses the proceeds to purchase capital. Individuals born at t therefore receive $(1+r_{t+1})\tau$ when they are old.
(a) Write down the individual's utility maximization problem, derive the first order condition and solve for savings. (10 points)
(b) Find the impact of this tax on savings (compute the corresponding derivative) and explain this effect. (10 points)
(c) Derive the per capita capital accumulation and find the corresponding steady state value. (10 points)
(d) Draw the corresponding diagram for the evolution of per capita capital if the initial per capita capital is smaller than the steady state value. (5 points)

Assignment 2

Q1.

The business cycle theory is used in an attempt to produce business cycles patterns that are consistent with major business cycle regularities observed in the data and also try to explain how much of the cyclical fluctuations in real output is due to productivity shocks.

The setup of the model:

- Time is discrete and the time horizon is infinite.
- Households:
  - The number of households is N, and they live forever.
  - Each household is endowed with 1 unit of time each period which is divided between work and leisure.
  - Households own an initial capital stock K_0, which is rent to firms and may augment through investment
  - Consumers are utility maximizing (determined in the model)
- Firms: there are a large number of identical firms who have access to technology, which combines labour and capital from households to produce the single good.
  - Firms are profit maximising
  - The firm's production function consists of a productivity shock component which is assumed to evolve exogenously (according to the law of motion).
  - Capital depreciates at the rate of δ.
- Timing of events:
  - ε_t is realized i.e. z_t is realized and observed to all parties.
  - Firms hire labour and rent capital from households to produce output, sell output and pay labour and capital.
  - Households divided their income into consumption and savings (holding capital). In making decision, households need to predict z_{t+1}.
  - This model has only one fluctuation – productivity shocks (a supply shock).
- Main assumptions of the Real Business Cycle theory are:
  - Al prices are flexible, even in the short-run
  - RBC models explicitly model consumers and firms' behaviour. Fluctuations in output, employment, and other variables are optimal responses of rational consumers and firms to exogenous shocks in economic environment (general equilibrium).
  - Early RBC models suggest that a large fraction of aggregate fluctuations could be understood as an efficient response to technology shocks that affected the entire economy, implying that there was little role for government stabilization policy.

The business cycle model is important in explaining how much of the cyclical fluctuations in real output is due to productivity shocks. Also, the most important contribution of the RBC development is it has initiated a prototype model (a stochastic neoclassical growth model) and a set of analytical tools for modern business cycle research.

Q2. The differences between RBC and the Keynesian model:

- Sources of fluctuations:
    - Keynesian: the main driving forces of fluctuations focuses on the effect of one-time change in aggregate demand, such as expectations about future profits, government purchases and money supply.
    - RBC: the driving force of the fluctuations comes from technology shocks which is a supply side shocks.
- Key Assumptions:
    - Keynesian: assumes that there are barriers to the instantaneous adjustment of nominal prices and wages which means that there is nominal stickiness in prices.
    - RBC: all prices are flexible in the RBC model, even in the short-run.
- Policy implications:
    - Keynesian: the fact that prices and wages are slow to move to clear markets implies that there is a role for monetary policy and fiscal policy in stability the economy in response to aggregate shocks.
    - RBC: a large fraction of fluctuations could be understood as an efficient response to technology shocks that affected the entire economy, implying there are little role for government stabilization policy.

Q3. Ramsey model

a)

- Representative household budget constraint in a given period t:

$$c_t + s_t = w_t A_t + (1 + r_t)s_{t-1}$$

$$s_{-1} = \frac{K_0}{L}$$

- Utility maximization problem:

$$\max_{\{s_t\}_{t=0}^{\infty}} U(c_0, c_1, \dots) = \sum_{t=0}^{\infty} \beta^t [\log(c_t) + \gamma \log(l_t)]$$

b) The equation that determines the consumption path

Individual chooses a sequence of savings of maximize his lifetime utility, subject to budget constraints in each period, taking as given prices and technology levels.

We only need to solve for $\{s_t\}_{t=0}^{\infty}$, and since only c_t and c_{t+1} consist of s_t, so only $\frac{\partial c_t}{\partial s_t}$ and $\frac{\partial c_{t+1}}{\partial s_t}$ are nonzero, where:

$$c_t = w_t A_t + (1 + r_t)s_{t-1} - s_t$$

$$c_{t+1} = w_{t+1} A_{t+1} + (1 + r_{t+1})s_t - s_{t+1}$$

Also, since l_t does is not a function of s_t, the l_t terms become zero after differentiated.

Differentiate the utility function U with respect to s_t:

$$\frac{\partial U}{\partial s_t} = \beta^t \frac{-1}{c_t} + \beta^{t+1} \frac{1 + r_{t+1}}{c_{t+1}} = 0$$

$\Longrightarrow \quad \frac{c_{t+1}}{c_t} = \beta\left(1 + r_{(t+1)}\right)$ – Euler equation (consumption path)

c) Conditions that characterize firms' profit maximization problem

- Markets are perfectly competitive, so the interest rate and wage rate are equal to their marginal product.

$$r_t = f'(k_t)$$

$$w_t = f(k_t) - f'(k_t)k_t$$

- Market clearing conditions:
  - Capital market: $K_{t+1} = L \cdot s_t$
  - Labour market: $L_t^D = L^s$

d) Derive the per capita capital accumulation for this economy

$$s_t = \frac{K_{t+1}}{L} = \frac{A_{t+1} L k_{t+1}}{L} = A_{t+1} k_{t+1} = (1 + g) A_t k_{t+1}$$

The production function is: $f(k_t) = k_t^\alpha$

The budget constraint is:

$$c_t + s_t = w_t A_t + (1+r_t)s_{t-1}$$

$$c_t + (1+g)A_t k_{t+1} = w_t A_t + (1+r_t)A_t k_t$$

$$c_t = A_t[w_t + (1+r_t)k_t - (1+g)k_{t+1}]$$

$$\tilde{c}_t = \frac{c_t}{A_t} = f(k_t) - f'(k_t)k_t + \left(1 + f'(k_t)\right)k_t - (1+g)k_{t+1}$$

$$\tilde{c}_t = f(k_t) + k_t - (1+g)k_{t+1}$$

$$\tilde{c}_t = k_t^\alpha + k_t - (1+g)k_{t+1} \tag{1}$$

Where $c_t = A_t \tilde{c}_t$

So the Euler equation becomes:

$$\frac{A_{t+1}\tilde{c}_{t+1}}{A_t \tilde{c}_t} = \beta(1 + r_{t+1})$$

$$\frac{\tilde{c}_{t+1}}{\tilde{c}_t} = \frac{\beta(1+r_{t+1})}{1+g} \tag{2}$$

e) Derive the steady-state

In the steady-state:

- $\tilde{c}_t = \tilde{c}_{t+1} = \tilde{c}^*$
- $k_t = k_{t+1} = k^*$

So from the transition equation above,

(1): $$\tilde{c}^* = {k^*}^\alpha + k^* - (1+g)k^*$$

$$\tilde{c}^* = {k^*}^\alpha - gk^* \tag{3}$$

(2): $$1 + g = \beta(1 + r_{t+1})$$

$$1 + g = \beta(1 + f'(k^*)$$

$$1 + g = \beta(1 + \alpha {k^*}^{\alpha-1})$$

$${k^*}^{\alpha-1} = \frac{1+g-\beta}{\alpha\beta}$$

So the steady state capital per capita for this economy is:

$$k^* = \left[\frac{1+g-\beta}{\alpha\beta}\right]^{\frac{1}{\alpha-1}}$$

So the steady state consumption per capita for this economy is:

$$\tilde{c}^* = \left[\frac{1+g-\beta}{\alpha\beta}\right]^{\frac{\alpha}{\alpha-1}} - g\left[\frac{1+g-\beta}{\alpha\beta}\right]^{\frac{1}{\alpha-1}}$$

Q4. The Diamond model

a) The individual's utility maximization problem

$$max_{\{c_{1t,}c_{2,t+1}\}}U$$

Where

- the utility function is $U = \log(c_{1t}) + \beta \log(c_{2t+1})$
- $c_{1t} + s_t = w_t A_t - \tau$
- $c_{2t+1} = (1 + r_{t+1})(s_t + \tau)$
- The budget constraint $c_{1t} + \frac{c_{2t+1}}{1+r_{t+1}} = w_t A_t$

The Lagrangian is:

$$\mathcal{L} = \log(c_t) + \beta \log(c_{2t+1}) + \lambda \left(w_t A_t - c_{1t} - \frac{c_{2t+1}}{1 + r_{t+1}} \right)$$

The first order conditions:

- $\frac{\partial \mathcal{L}}{\partial c_{1t}} = \frac{1}{c_t} = \lambda$
- $\frac{\partial \mathcal{L}}{\partial c_{2t+1}} = \frac{\beta}{c_{t+1}} = \frac{\lambda}{1+r_{t+1}}$

$\Longrightarrow c_{1t} = \frac{c_{2t+1}}{\beta(1+r_{t+1})}$ (1)

Substitute (1) into the budget constraint:

$$\frac{c_{2t+1}}{\beta(1 + r_{t+1})} + \frac{c_{2t+1}}{1 + r_{t+1}} = w_t A_t$$

$$c_{2t+1} = \frac{\beta}{1 + \beta} w_t A_t (1 + r_{t+1})$$

Substitute c_{2t+1} into (1):

$$c_{1t} = \frac{w_t A_t}{1+\beta} \qquad (2)$$

Since

$$s_t = w_t A_t - \tau - c_{1t}$$

$$s_t = w_t A_t - \frac{w_t A_t}{1 + \beta} - \tau$$

So the savings function for this economy is

$$s_t = \frac{\beta}{1 + \beta} w_t A_t - \tau$$

b) The impact of tax on saving

$$\frac{\partial s_t}{\partial \tau} = -1$$

The derivative of saving w.r.t tax is -1.

This implies that saving and tax is perfectly negatively related. Hence, savings will decrease by the same amount as the tax increase. In this economy, the tax is collected when an individual is young and paid back when the individual becomes old with interest. So, in a sense, the tax here is like a dissaving imposed by the government. Therefore with a tax of 10%, people who want to save 20% of their income should now save 10% instead and they will receive 10% saving plus 10% tax paid back by the government when they are old. Overall, tax has no effect on consumption and savings decision in this case.

c) Steady-state per capita capital

From part b) above, we should treat this problem as there is no tax system since it has no effect on consumption and savings decisions.

So the problem becomes:

$$K_{t+1} = Ls_t$$

$$K_{t+1} = L\left(\frac{\beta}{1+\beta} w_t A_t\right)$$

Where $w_t = f(k_t) - f'(k_t)k_t = (1-\alpha)k^\alpha$

Hence,

$$K_{t+1} = L\left[\frac{\beta}{1+\beta}(1-\alpha)k_t^\alpha A_t\right]$$

Since $A_{t+1} = A_t$ and $L_{t+1} = L_t = L$, divide both side by $A_{t+1}L_{t+1}$

$$k_{t+1} = \frac{\beta}{1+\beta}(1-\alpha)k_t^\alpha$$

In the steady state $k_t = k_{t+1} = k^*$ hence,

$$k^* = \frac{\beta}{1+\beta}(1-\alpha)k^{*\alpha}$$

$$k^{*1-\alpha} = \frac{\beta}{1+\beta}(1-\alpha)$$

Hence the steady state per capita capital is

$$k^* = \left[\frac{\beta(1-\alpha)}{1+\beta}\right]^{\frac{1}{1-\alpha}}$$

d) The diagram for the evolution of per capita capital if the initial per capita capital is smaller than the steady state value